

# PageRank: Power Iteration Animations

This page animates the iteration  $(x_{n+1}) = Qx_n$  on the 6-node PageRank example from Day 5.

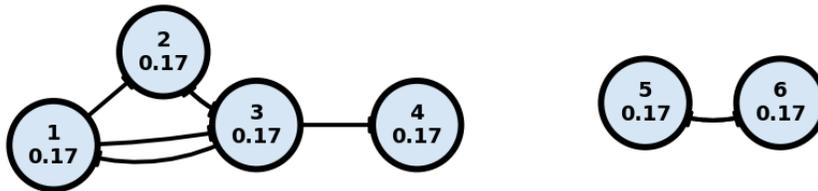
- **Node fill:** value of  $(x_n)$  at that node.
- **Labels:** node index and  $(x_n)$  (rounded).
- **Note:** In the **Raw (Q)** case, the dangling node (4) causes loss of total mass; this is visible via  $\text{sum}(x)$  in the frame title.

## Raw (Q) (dangling node)

$x_0$ : uniform

$$x_0 = (1/6)\mathbf{1}$$

Raw Q (dangling node)  $x_0 = (1/6)\mathbf{1}$  step 0

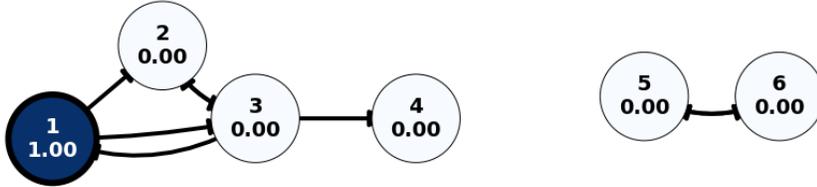


$$Q = \begin{bmatrix} 0 & 0 & 1/3 & 0 & 0 & 0 \\ 1/2 & 0 & 1/3 & 0 & 0 & 0 \\ 1/2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \quad x_0 = \begin{bmatrix} 1/6 \\ 1/6 \\ 1/6 \\ 1/6 \\ 1/6 \\ 1/6 \end{bmatrix}$$

$x_0: e_1$

$x_0 = e_1$

Raw Q (dangling node)  $x_0 = e_1$  step 0

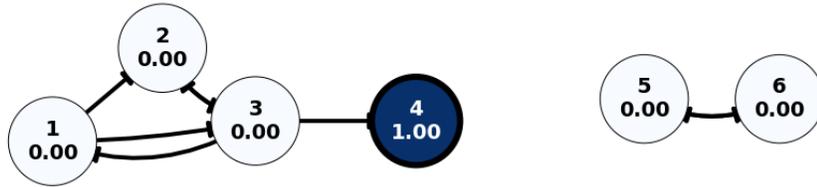


$$Q = \begin{bmatrix} 0 & 0 & 1/3 & 0 & 0 & 0 \\ 1/2 & 0 & 1/3 & 0 & 0 & 0 \\ 1/2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \quad x_0 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_0: e_4$

$x_0 = e_4$

Raw Q (dangling node)  $x_0 = e_4$  step 0

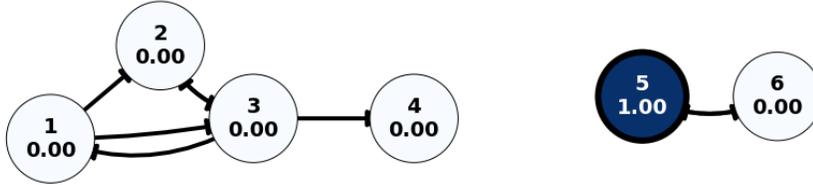


$$Q = \begin{bmatrix} 0 & 0 & 1/3 & 0 & 0 & 0 \\ 1/2 & 0 & 1/3 & 0 & 0 & 0 \\ 1/2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \quad x_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$x_0: e_5$

$x_0 = e_5$

Raw Q (dangling node)  $x_0 = e_5$  step 0



$$Q = \begin{bmatrix} 0 & 0 & 1/3 & 0 & 0 & 0 \\ 1/2 & 0 & 1/3 & 0 & 0 & 0 \\ 1/2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \quad x_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

### Final States (Raw (Q))

Final states after 30 iterations for each starting vector:

Raw Q (dangling node)  $x_0 = (1,6)/11$  step 30



Uniform:

Raw Q (dangling node)  $x_0 = e_1$  step 30



$e_1$ :

Raw Q (dangling node)  $x_0 = e_4$  step 30



$e_4$ :

Raw Q (dangling node)  $x_0 = e_5$  step 30



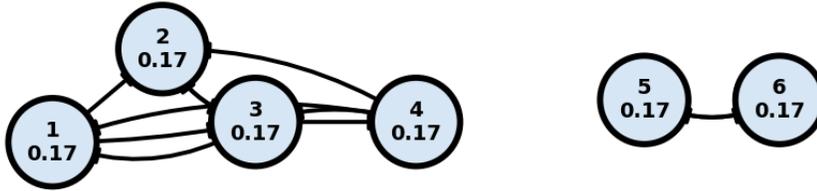
$e_5$ :

## Corrected (P) (dangling fixed)

$x_0$ : uniform

$$x_0 = (1/6)\mathbf{1}$$

Corrected P (dangling fixed)  $x_0 = (1/6)\mathbf{1}$  step 0

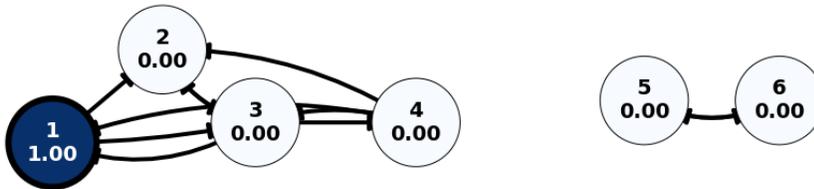


$$P = \begin{bmatrix} 0 & 0 & 1/3 & 1/3 & 0 & 0 \\ 1/2 & 0 & 1/3 & 1/3 & 0 & 0 \\ 1/2 & 1 & 0 & 1/3 & 0 & 0 \\ 0 & 0 & 1/3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \quad x_0 = \begin{bmatrix} 1/6 \\ 1/6 \\ 1/6 \\ 1/6 \\ 1/6 \\ 1/6 \end{bmatrix}$$

$x_0$ :  $e_1$

$$x_0 = e_1$$

Corrected P (dangling fixed)  $x_0 = e_1$  step 0

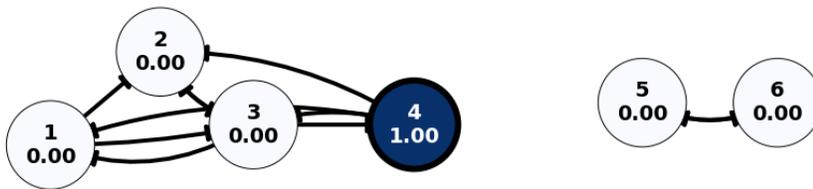


$$P = \begin{bmatrix} 0 & 0 & 1/3 & 1/3 & 0 & 0 \\ 1/2 & 0 & 1/3 & 1/3 & 0 & 0 \\ 1/2 & 1 & 0 & 1/3 & 0 & 0 \\ 0 & 0 & 1/3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \quad x_0 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

**$x_0: e_4$**

$$x_0 = e_4$$

Corrected P (dangling fixed)  $x_0 = e_4$  step 0

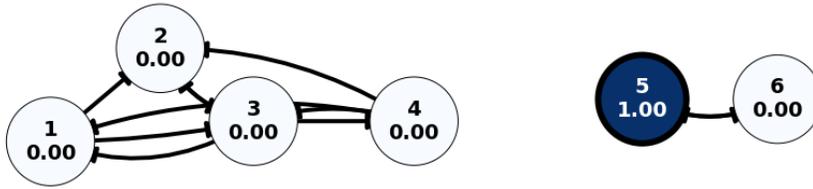


$$P = \begin{bmatrix} 0 & 0 & 1/3 & 1/3 & 0 & 0 \\ 1/2 & 0 & 1/3 & 1/3 & 0 & 0 \\ 1/2 & 1 & 0 & 1/3 & 0 & 0 \\ 0 & 0 & 1/3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \quad x_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

**$x_0: e_5$**

$$x_0 = e_5$$

Corrected P (dangling fixed)  $x_0 = e_5$  step 0



$$P = \begin{bmatrix} 0 & 0 & 1/3 & 1/3 & 0 & 0 \\ 1/2 & 0 & 1/3 & 1/3 & 0 & 0 \\ 1/2 & 1 & 0 & 1/3 & 0 & 0 \\ 0 & 0 & 1/3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \quad x_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

### Final States (Corrected (P))

Final states after 30 iterations for each starting vector:

Corrected P (dangling fixed)  $x_0 = (1/6)1$  step 30



Uniform:

Corrected P (dangling fixed)  $x_0 = e_1$  step 30



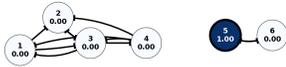
$e_1$ :

Corrected P (dangling fixed)  $x_0 = e_4$  step 30



$e_4$ :

Corrected P (dangling fixed)  $x_0 = e_5$  step 30



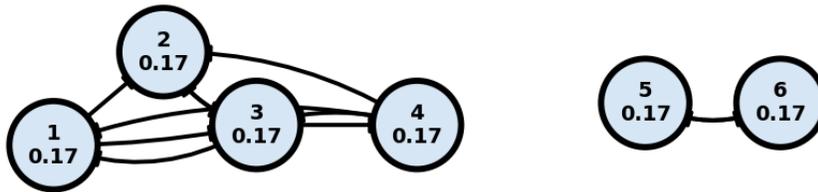
$e_5$ :

## Teleporting ( $G$ ) ( $\alpha=0.85$ )

$x_0$ : uniform

$$x_0 = (1/6)\mathbf{1}$$

Teleporting  $G$  ( $\alpha=0.85$ )  $x_0 = (1/6)\mathbf{1}$  step 0

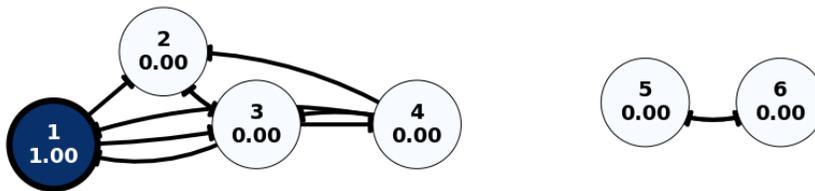


$$G = 0.85P + 0.15\frac{1}{6}\mathbf{1}\mathbf{1}^T = 0.85P + 0.025 \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}, \quad x_0 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_0$ :  $e_1$

$$x_0 = e_1$$

Teleporting  $G$  ( $\alpha=0.85$ )  $x_0 = e_1$  step 0

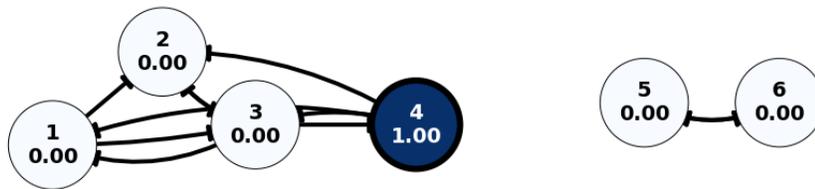


$$G = 0.85P + 0.15\frac{1}{6}\mathbf{1}\mathbf{1}^T = 0.85P + 0.025 \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}, \quad x_0 = \begin{bmatrix} 1/6 \\ 1/6 \\ 1/6 \\ 1/6 \\ 1/6 \\ 1/6 \end{bmatrix}$$

**$x_0: e_4$**

$$x_0 = e_4$$

Teleporting G (alpha=0.85)  $x_0 = e_4$  step 0

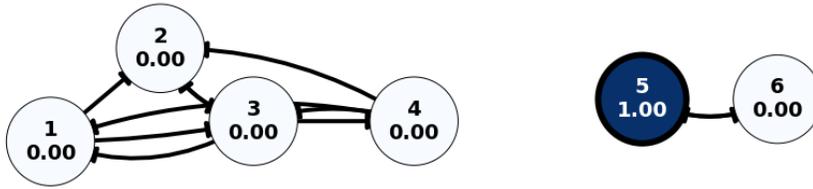


$$G = 0.85P + 0.15\frac{1}{6}\mathbf{1}\mathbf{1}^T = 0.85P + 0.025 \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}, \quad x_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

**$x_0: e_5$**

$$x_0 = e_5$$

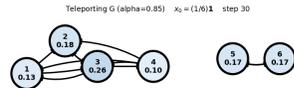
Teleporting G (alpha=0.85)  $x_0 = e_5$  step 0



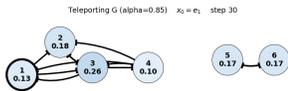
$$G = 0.85P + 0.15\frac{1}{6}\mathbf{1}\mathbf{1}^T = 0.85P + 0.025 \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}, \quad x_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

### Final States (Teleporting (G))

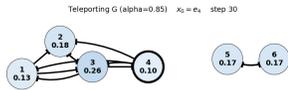
Final states after 30 iterations for each starting vector:



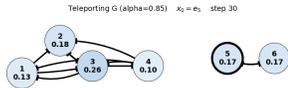
Uniform:



$e_1$ :



$e_4$ :

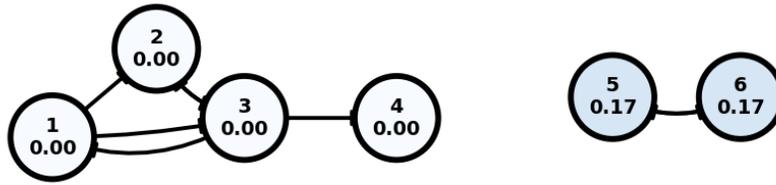


$e_5$ :

### All Final States Comparison

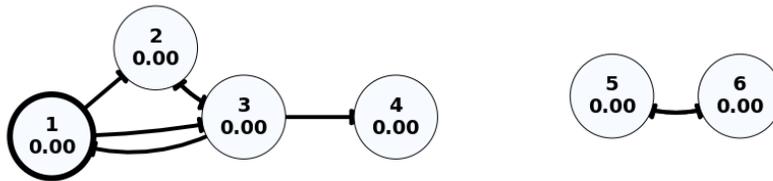
Raw (Q):

Raw Q (dangling node)  $x_0 = (1/6)\mathbf{1}$  step 30



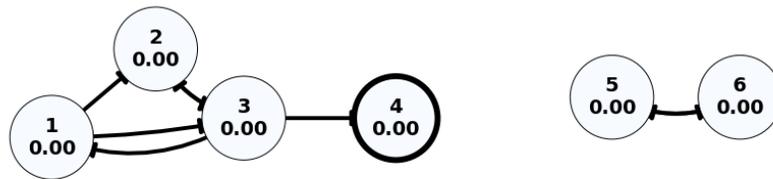
Uniform:

Raw Q (dangling node)  $x_0 = e_1$  step 30



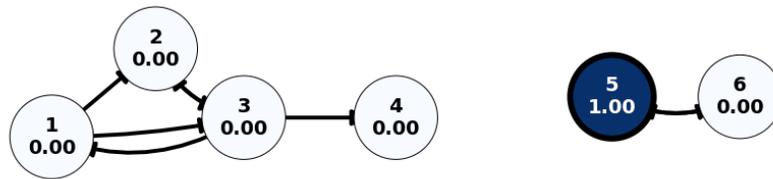
$e_1$ :

Raw Q (dangling node)  $x_0 = e_4$  step 30



$e_4$ :

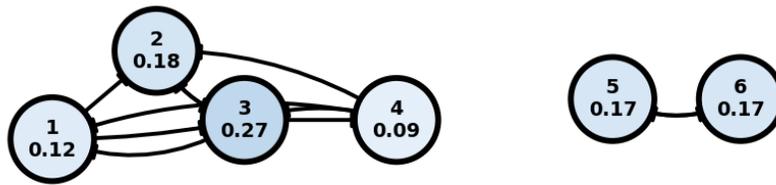
Raw Q (dangling node)  $x_0 = e_5$  step 30



$e_5$ :

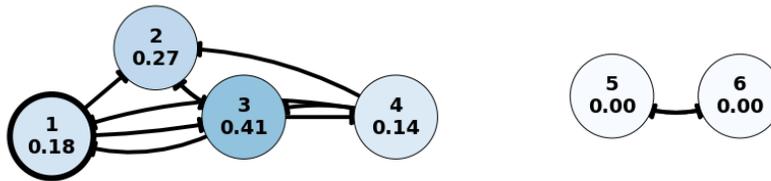
Corrected (P):

Corrected P (dangling fixed)  $x_0 = (1/6)\mathbf{1}$  step 30



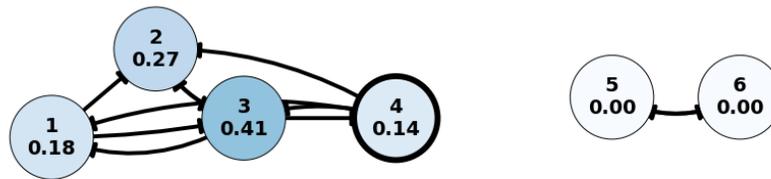
Uniform:

Corrected P (dangling fixed)  $x_0 = e_1$  step 30



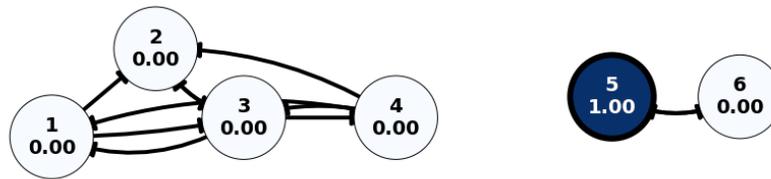
$e_1$ :

Corrected P (dangling fixed)  $x_0 = e_4$  step 30



$e_4$ :

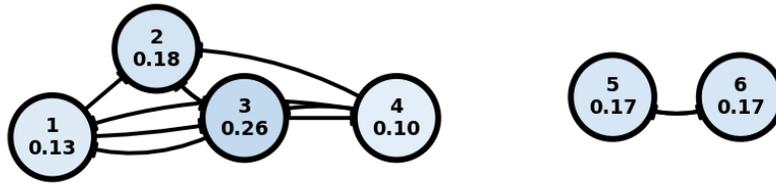
Corrected P (dangling fixed)  $x_0 = e_5$  step 30



$e_5$ :

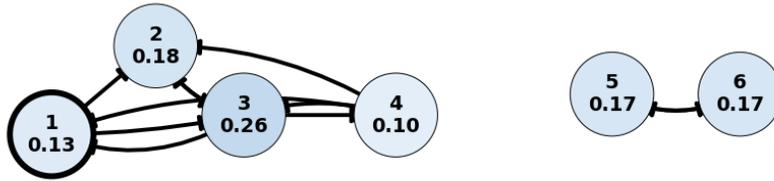
Teleporting (G):

Teleporting G (alpha=0.85)  $x_0 = (1/6)\mathbf{1}$  step 30



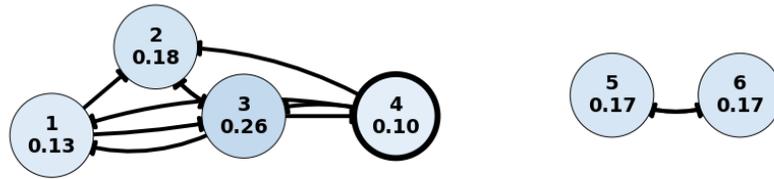
Uniform:

Teleporting G (alpha=0.85)  $x_0 = e_1$  step 30



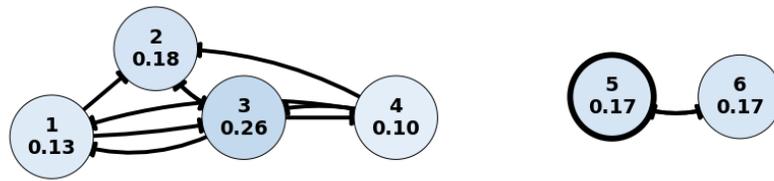
$e_1$ :

Teleporting G (alpha=0.85)  $x_0 = e_4$  step 30



$e_4$ :

Teleporting G (alpha=0.85)  $x_0 = e_5$  step 30



$e_5$ :